HYDRODYNAMIC DEVELOPMENT OF A LAMINAR VELOCITY FIELD IN RECTANGULAR CHANNELS

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Test data are shown on the development of a laminar flow in the entrance of rectangular channels with the ratio of sides 2 : 1, 4 : 1, and 10 : 1.

The flow of a viscous fluid in the entrance zone of a channel has been analyzed rather extensively in the Soviet Union as well as in other countries. All these studies pertain essentially to the development of a flow field in the entrance zone of a circular pipe [1-3] or of a flat channel [4-6]. Only a few of these studies [7] have also dealt more thoroughly with the flow in the entrance zone of rectangular channels.

While much theoretical material is available, relatively little experimental material has been published so far [8].

The authors of this report tested rectangular channels with the ratio of sides 2 : 1, 4 : 1, and 10 : 1, the Reynolds number varying from 200 to 600. The channels were made of polished acrylic glass, with the section dimensions (B × A) equal to 2 × 1, 4 × 1, and 10 × 1 cm, respectively. Windows made of optical glass were mounted in the lateral walls. The active fluid was distilled water.

The velocity profile was measured by the visualization method, with a metallic wire inserted into the stream and heated by short electric discharge pulses [9]. A Nichrome wire 0.1 mm in diameter served this purpose, being placed on the stream axis successively at various distances from the entrance section of a hydraulic channel.

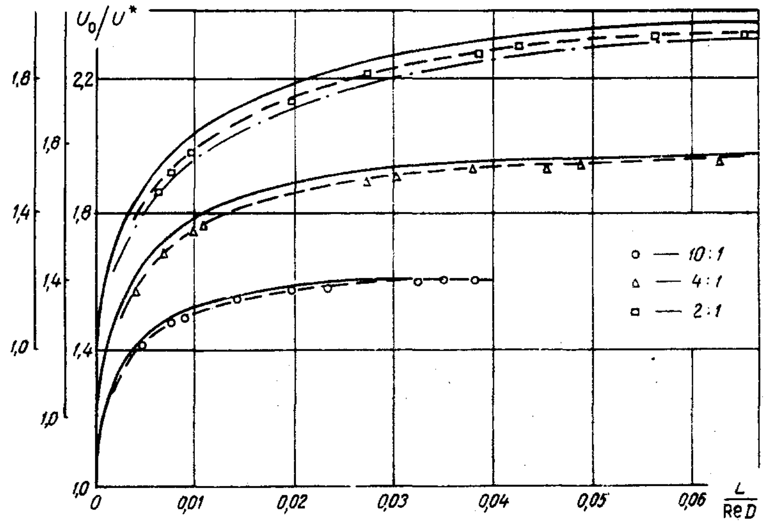

Fig. 1. Referred length of the entrance zone in a channel, as a function of the ratio of axial velocity to mean velocity.

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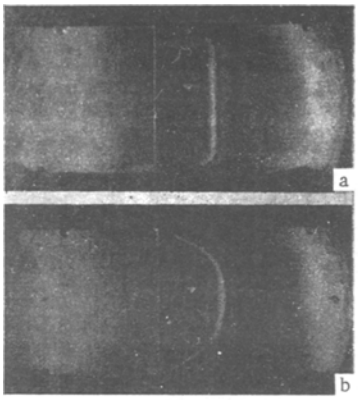

Fig. 2

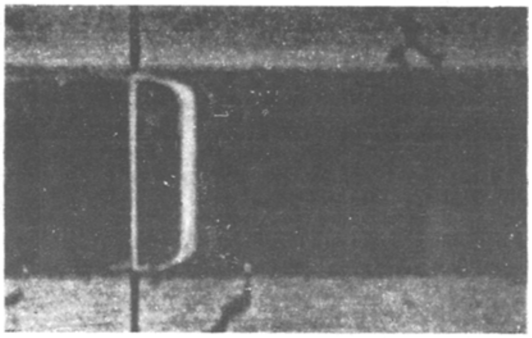

Fig. 3

Fig. 2. Successive development of the velocity profile in the 2 : 1 channel.

Fig. 3. Fully developed velocity profile in the 10 : 1 channel, measured along one of the wide walls.

Variations in the axial velocity were recorded along the longitudinal channel coordinate and the length of the entrance zone was determined on this basis. The mean velocity was determined from the water volume discharged at the channel exit during a given interval of time. The axial velocity was determined from the velocity of a few successive fronts produced near the wire as a result of electric pulses applied to the latter in sequence at equal time intervals. The correction to this velocity, to account for the velocity defect behind the wire, was based on the empirical relation [10]

$$\frac{U_0-U}{U_0}=1.514\left(\frac{x}{d}\right)^{-0.48}. \tag{1}$$

The test results are shown in Fig. 1 in terms of the referred channel length as a function of the ratio of axial velocity to mean velocity. For convenience, the U_0/U^* scales for the three different channels have been shifted from one another. The solid lines on the diagram represent the analytical solution in [7], the dashed lines represent our analytical solution, and the dashed-dotted lines for the 2 : 1 channel represent the test data in [8]. Since there was no analytical solution available for the 10 : 1 channel, the ordinates U_0/U^* were for this case found by extrapolation.

An examination of the test data has revealed that in channels with the ratio of sides 10 : 1 or 4 : 1 the flow develops sooner than in a channel with the ratio of sides 2 : 1, where the flow is fully developed only at $L/\mathrm{Re}\,D \geq 0.09$. In 10 : 1 and in 4 : 1 channels the flow is fully developed at $L/\mathrm{Re}\,D \geq 0.035$ or 0.07, respectively, i.e., the calculated values of velocity at the axis are higher than its measured values.

In all three cases did the test points lie below the theoretical values, especially near the entrance section of a channel. Moreover, the agreement between calculated and measured values was found most satisfactory for the channels with the ratio of sides 10 : 1 and 4 : 1. It is to be noted that each test point in Fig. 1 represents the root-mean-square of five or six readings obtained at distances approximately (80 to 250)x/d from the wire. The values corresponding to full development in the test channels were, respectively, 1.985, 1.765, and 1.600 as compared with the theoretical values 1.993, 1.773, and 1.605 in [7].

For illustration, we show in Fig. 2 the photographs of successive stages of developing velocity profiles, measured along one wide wall of the 2 : 1 channel at two distances $L/\mathrm{Re}\,D = 0.00263$ (a) and 0.0124 (b).

Since no direct data on the velocity profile are given in [7], it is not possible to compare theoretically and experimentally established laws of velocity distribution within the zone where the flow field is developing. Such a comparison can only be made for the zone where the velocity field has fully developed and for which calculated data are available. In Fig. 3 is shown the photograph of a fully developed velocity profile in the 10 : 1 channel, measured along one of the wide channel walls. Points along this profile corresponding to the exact analytical solution in [11]

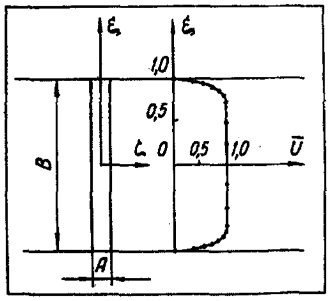

Fig. 4

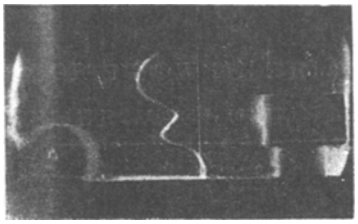

Fig. 5

Fig. 4. Comparison between the measured and the calculated velocity profile in the 10 : 1 channel.

Fig. 5. Velocity profile in the wake behind two crossing hollow cylinders.

$$\bar{U}=\frac{\sum\limits_{k=1,3,5\ldots}^{\infty}(-1)^{\frac{1}{2}(k-1)}k^{-3}\cos\left(\frac{1}{2}k\pi\zeta\right)\left\{1-\frac{\cos h\left(\frac{1}{2}k\pi\xi\frac{B}{A}\right)}{\cos h\left(\frac{1}{2}k\pi\frac{B}{A}\right)}\right\}}{\sum\limits_{k=1,3,5\ldots}^{\infty}(-1)^{\frac{1}{2}(k-1)}k^{-3}\left\{1-\frac{1}{\cos h\left(\frac{1}{2}k\pi\frac{B}{A}\right)}\right\}}\ldots \tag{2}$$

are plotted in Fig. 4. For convenience, Figs. 3 and 4 are both drawn to the same scale. The agreement between calculations and test results is very close. However, Eq. (2) is unwieldy and the more convenient approximate solution in [12]

$$\bar{U}=(1-|\xi|^m)(1-|\zeta|^n)\ldots, \tag{3}$$

with $m \simeq 2.3$ for $0.67 \leq A/B \leq 1.0$, $m \simeq 1.54\, B/A$ for $0 \leq A/B \leq 0.67$, and $n \simeq (2.0$ to $2.3)$ has been found to yield an entirely satisfactory description of a fully developed velocity profile. The velocity profile along the axis parallel to the narrow sides in channels with the ratio of sides 10 : 1 and 4 : 1, for example, is accurately enough described by a second-degree parabola. It is to be noted that Eq. (3), where the exponents m and n depend on the ratio of sides, is more accurate near the walls than toward the center of a channel.

In conclusion, we note that this method is recommended for the study of complex flow modes and for the description of velocity fields which are difficult to analyze mathematically. As an example of such a flow pattern, we show in Fig. 5 the photograph of the velocity field in the wake behind two hollow cylinders inside a closed volume. The backstream zone behind the transversely oriented cylinder is distinctly evident in the lower part of the picture.

NOTATION

A	is the channel height;
B	is the channel width;
L	is the distance from the entrance section to the point of measurement;
Re	is the Reynolds number referred to the equivalent channel diameter;
D	is the equivalent diameter of a channel;
U_0	is the true velocity at the channel axis;
U	is the measured velocity at the channel axis;
U*	is the mean velocity;
x	is the axial distance from the wire to the point of measurement;
d	is the wire diameter;
$\bar{U}$	is the ratio of instantaneous velocity to maximum axial velocity;
$\zeta = 2z/A$,	
$\xi = 2y/B$	are the referred channel coordinates;
m, n	are the exponents in Eq. (3).

LITERATURE CITED

1. L. Schiller, Zeitschr. für Angew. Mathem. und Mech., 2, 96 (1922).
2. H. L. Langhaar, J. Appl. Mech., 42 (1942).
3. B. Atkinson, M. P. Brocklebank, C. C. H. Card, and L. M. Smith, AIChEJ, 15, No. 4, 548-555 (1969).
4. I. N. Sadikov, Inzh.-Fiz. Zh., 12, No. 2 (1967).
5. N. I. Buleev, Prikl. Mekh. i Tekh. Fiz., 3 (1967).
6. G. Schlichting, Theory of the Boundary Layer [Russian translation], Nauka, Moscow (1969).
7. L. S. Han, J. Appl. Mech., 27, 403-409 (1960).
8. E. M. Sparrow, C. W. Hixon, and G. J. Shavit, Basic Engrg., 87, No. 1, 131-140 (1967).
9. S. D. Solomonov and G. G. Spirin, Prikl. Mekh. i Tekh. Fiz., 5 (1970).
10. S. D. Solomonov and A. R. Gordon, Proceedings of the Fourth All-Union Conference on Heat and Mass Transfer. Heat and Mass Transfer [in Russian], Vol. 1, Pt. 1, Minsk (1972).
11. D. B. Holmes and J. R. Vermenlen, Chem. Engrg. Sci., 23, No. 7, 717-722 (1968).
12. H. F. P. Purday, Streamline Flow, 16-18 (1949).